Recent developments in the theory of boundary value problems factorization.

J. Henry,
INRIA Bordeaux Sud ouest, France

The theory of boundary value problems factorization for linear elliptic problems has been developped in collaboration with A. Ramos (Madrid), B. Louro, C. Soares, M. Orey (Lisbon). The simplest presentation refers to the Poisson equation in a cylinder. We consider the following problem

$$(\mathcal{P}_0)\begin{cases} -\Delta u = f \quad \text{in } \Omega, f \in L^2(\Omega) \\ u|_\Sigma = 0, \quad -\dfrac{\partial u}{\partial n}|_{\Gamma_0} = u_0 \in (H^{1/2}_{0,0}(\mathcal{O}))' \quad u|_{\Gamma_a} = u_1 \in H^{1/2}_{0,0}(\mathcal{O}) \end{cases}$$

where Ω is the cylinder $\mathcal{O}\times]0,a[$, $\Gamma_0 = \mathcal{O}\times\{0\}$, $\Gamma_a = \mathcal{O}\times\{a\}$, $\Sigma = \mathcal{O}\times\partial\mathcal{O}$. In that case the factorized form reads

$$\begin{cases} \frac{dP}{dx} + P^2 + \Delta_y = 0; & P(0) = O \\ \frac{dw}{dx} + Pw = -f; & w(0) = u_0 \\ -\frac{du}{dx} + Pu = -w; & u(a) = u_1. \end{cases}$$

In this talk we will present various extensions of this method: extension to the Stokes equation (F. Jday thesis, Tunis). We will present formal extension to a non linear problem and to a parabolic problem. We will show the use of the factorization for the solution of the ill-posed Cauchy problem for the laplacian (F. Jday).