A class of singular first order ODE's with applications in reaction-diffusion

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Abstract

We study positive solutions $y(u)$ for the first order differential equation

$$y' = q(c\, y^{\frac{1}{p}} - f(u))$$

where $c > 0$ is a parameter, $p > 1$ and $q > 1$ are conjugate numbers and f is a continuous function in $[0, 1]$ such that $f(0) = 0 = f(1)$. We shall be particularly concerned with positive solutions $y(u)$ such that $y(0) = 0 = y(1)$. Our motivation lies in the fact that this problem provides a model for the existence of travelling wave solutions for analogues of the FKPP equation in one space dimension, where diffusion is represented by the p-Laplacian operator. We obtain a theory of admissible velocities and some other features that generalize classical and recent results, established for $p = 2$.