In this talk I will develop a duality theory for classical problems of the Calculus of Variations of the kind

$$J(\Omega) := \inf\left\{\int_\Omega (f(\nabla u) + g(u))\,dx + \int_{\Gamma_1} \gamma(u)\,dH^{d-1}\ ,\ u = 0 \text{ on } \Gamma_0\right\}$$

where g, γ are possibly non convex functions with suitable growth conditions and f is a convex intergrand on $\mathbb{R}^d$. Here (Γ_0, Γ_1) is a partition of $\partial\Omega$. A challenging issue is to characterize the global minimizers of such a problem and the stability of the minimal value (with respect for instance to small deformations of the domain Ω).

We present a duality scheme in which the dual problem reads quite nicely as a linear programming problem. The solvability of this dual problem is a major issue. It can be achieved in the one dimensional case and in higher dimensions under special assumptions on f, g. Applications are given for a class of free boundary problems.