All Triangles at Once

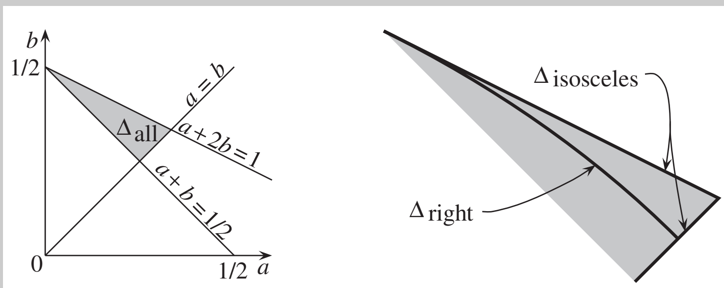
A triangle is determined modulo similarity. This is done by reordering the sides a, b, and c so that $a \le b \le c$ and rescaling so that the perimeter $a + b + c$ is 1, where $(a, b) \in \mathbb{R}^2$ such that

$$\left.\begin{array}{r} 0 \le a \le b \le c \\ a+b+c=1 \\ a \le b+c,\ b \le a+c,\ c \le a+b \end{array}\right\} \quad \Leftrightarrow \quad (*) \begin{cases} 0 \le a \le b \\ a+2b \le 1 \\ 1/2 \le a+b \\ c = 1-a-b \end{cases}.$$

The set $\Delta_{\text{all}} = \{(a, b) \in \mathbb{R}^2 : (*)\}$ of all "triangles" is itself a triangle. We invite the reader to find Δ_{acute}, Δ_{obtuse}, and $\Delta_{\text{equilateral}}$. We see below where the following classes of triangles lie in Δ_{all}:

$$\Delta_{\text{right}} = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 = c^2\} \quad = \{(a, b) \in \mathbb{R}^2 : a + b - ab = 1/2\},$$

$$\Delta_{\text{isosceles}} = \{(a, b) \in \mathbb{R}^2 : a = b \vee b = c\} = \{(a, b) \in \mathbb{R}^2 : a = b \vee a + 2b = 1\}.$$



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